This thesis deals with the study of a particular, stochastic and astrophysical background of gravitational waves. An astrophysical background is produced by a superposition of gravitational waves generated by statistically independent events. In this work we considered hyperbolic encounters between two black holes, occurring at a high rate within particularly dense areas of Galactic halos.

First, we defined some key concepts employed in this field. Then we showed a method meant to calculate the spectrum of the energy carried by such a background. We initially took into account only the linear contribution to the Einstein equations.

Next, we introduced the "memory" effect. This effect appears when higher-order contributions are considered during the resolution of the Einstein equations. Consequently, its determination involves the use of post-Newtonian expansions.

The “memory” effect entails an additional contribution to the energy spectrum at lower frequencies. It is named after the different metric’s properties within the phase prior to the onset of the gravitational wave emission, and the one that follows the latter shut down.

This effect is heavily dependent on the system’s evolutionary history. In fact, past events provide larger contributions than the more recent ones. Afterward, we evaluated the influence exerted by the “memory” effect on the background spectrum.

Finally, we presented an algorithm able to invert the relationship correlating the spectral density of the energy, and black holes’ number density per unit mass.

This inversion will allow getting some information about the numerical density, since it is assumed that the spectral density will be directly observed in the next future.

A numerical approach was used for the inversion. This is based on the discretization of the involved functions, as well as on both the use of minimization procedures and the singular value decomposition technique.